Worksheet 3.2—Rolle’s Theorem and the MVT

Show all work. No calculator unless otherwise stated.

Multiple Choice

_____ 1. Determine if the function \( f(x) = x\sqrt{6-x} \) satisfies the hypothesis of Rolle’s Theorem on the interval \([0,6]\), and if it does, find all numbers \(c\) satisfying the conclusion of that theorem.
   \[\text{(A) 2, 3} \quad \text{(B) 4, 5} \quad \text{(C) 5} \quad \text{(D) 4} \quad \text{(E) hypothesis not satisfied}\]

_____ 4. Determine if the function \( f(x) = x + x^{2/3} (1-x)^{1/3} \) satisfies the hypothesis of the MVT on \([0,1]\).
   If it does, find all possible values of \(c\) satisfying the conclusion of the MVT. (You will have to factor out least powers.)
   \[\text{(A) } \frac{2}{3} \]
   \[\text{(B) } \frac{1}{4} \]
   \[\text{(C) } \frac{1}{2} \]
   \[\text{(D) } \frac{1}{3} \]
   \[\text{(E) hypothesis not satisfied}\]

_____ 5. Which of the following functions below satisfy the hypothesis of the MVT?
   I. \( f(x) = \frac{1}{x+1} \) on \([0,2]\)
   II. \( f(x) = x^{1/3} \) on \([0,1]\)
   III. \( f(x) = |x| \) on \([-1,1]\)
   \[\text{(A) I only} \quad \text{(B) I and II only} \quad \text{(C) I and III only} \quad \text{(D) II only} \quad \text{(E) II and III only}\]

15. Suppose that we know that \( f(x) \) is continuous and differentiable on \([6,15]\). Let’s also suppose that we know that \( f(6) = -2 \) and that \( f'(x) \leq 10 \) for all \( x \in [6,15]\). What is the largest possible value for \( f(15) \)?
16. Let \( f(x) = \tan x \). Show that \( f(\pi) = f(2\pi) \) but that there is not number \( c \in (\pi, 2\pi) \) such that \( f'(c) = 0 \). Why does this not contradict Rolle’s Theorem?

11. (Calculator permitted) For \( f(x) = -x^4 + 4x^3 + 8x^2 + 5 \) on \([0, 5]\)
   (a) Determine if the MVT can be applied on the given interval. If so, find the value(s) guaranteed by the theorem.

   (b) Find the equation of the secant line on \([0, 5]\)

   (c) Find the equation of the tangent line at any value of \( c \) found above.

   (d) On your calculator, sketch a graph of \( f(x) \) on \([0, 5]\) along with the secant and tangent line(s). Sketch the graph below.

13. The function \( f(x) = \begin{cases} 0, & x = 0 \\ 1-x, & 0 < x \leq 1 \end{cases} \) is differentiable on \((0,1)\) and satisfies \( f(0) = f(1) \). However, its derivative is never zero on \((0,1)\). Does this contradict the Mean Value Theorem? Explain why or why not.

14. Determine the values of \( a, b, \) and \( c \) such that the function \( f \) satisfies the hypothesis of the MVT on the interval \([0,3]\).

   \[
   f(x) = \begin{cases} 
   1, & x = 0 \\
   ax + b, & 0 < x \leq 1 \\
   x^2 + 4x + c, & 1 < x \leq 3
   \end{cases}
   \]